Solution by P. H. PHILBRICK, C. E., Lake Charles, La.

The number of board feet in a log d inches in diameter and l inches in length is $n=\dfrac{\frac{1}{4}\pi d^2 l}{144}$.

If l is in feet, $n=\dfrac{\frac{1}{4}\pi d^2 l}{12}$.

Substituting $\frac{22}{7}$ for π and 10 for l we have, $n=\frac{5}{6}.\frac{11}{14}d^2$.

Allowing $\frac{1}{5}$ for saw cut, $n=\frac{4}{5}.\frac{5}{6}.\frac{11}{14}d^2=\frac{11}{21}d^2$.

Allowing $\frac{1}{2}$ inch for bark, $n=\frac{11}{21}(d-1)^2$.

For $d=22$, an average value, $(d-1)^2=\frac{441}{440}(d^2-2d)$.

$\therefore\ n=\frac{11}{21}.\frac{441}{440}(d^2-2d)=\frac{21}{40}(d^2-2d)$.

I would propose the formula $n=d^2$ (or $n=(d-1)^2$) for a log 20 feet long, since it is as accurate and much more simple than Wentworth's.

The most accurate formula, however, must be based on the end diameters of the log.

Let d and D represent those diameters.

Board feet in total volume of log 20 feet long

$$=\tfrac{1}{4}\pi.\tfrac{5}{3}\times\frac{d^2+dD+D^2}{3}=\tfrac{5}{9}\times\tfrac{11}{14}(d^2+dD+D^2).$$

(See Philbrick's *Engineer's Manual*, table 23).

Since $(D-d)^2=D^2-2dD+d^2>0$, $D^2+dD+d^2>3dD$.

Hence volume$>\frac{5}{3}\times\frac{11}{14}dD$.

Allowing $\frac{1}{5}$ for saw cut, $n>\frac{4}{3}\times\frac{11}{14}dD=\frac{22}{21}dD$.

Allowing $\frac{1}{22}$ of the above for bark we still have $n>dD$, or, say, $n=dD$ ....(1).

It is easily shown that volume$>\frac{5}{3}\times\frac{11}{14}\left[\dfrac{d+D}{2}\right]^2$

and as before that $n=\left[\dfrac{d+D}{2}\right]^2$....(2).

The author's experience leads him to believe that the above formulas are quite accurate, but that logs will cut a little more into large timbers than the formulas give.

If thought to give too large a result, in extreme cases, we might suggest the formula, $n=d(D-2)$....(3), or $n=\left[\dfrac{d+D}{2}-1\right]^2$....(4).

At all events the forms suggested should be used.

ALGEBRA.

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111. Proposed by ARTEMAS MARTIN, A. M., Ph. D., LL.D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Solve the equation $x(y+z)=a(x+y+z)$, $y(x+z)=b(x+y+z)$, $z(x+y)=c(x+y+z)$.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa., and HON. JOSIAH H. DRUMMOND, LL. D., Portland, Me.

Let $x+y+z=s$. Then (1)+(2)+(3) gives $xy+xz+yz=\frac{1}{2}(a+b+c)s$..(4).
(4)−(1) gives $yz=\frac{1}{2}(b+c-a)s$....(5).
(4)−(2) gives $xz=\frac{1}{2}(a+c-b)s$....(6).
(4)−(3) gives $xy=\frac{1}{2}(a+b-c)s$....(7).
(6)÷(7) gives $z/y=(a+c-b)/(a+b-c)$....(8).

(8) multiplied by (5) gives $z=\sqrt{\dfrac{(a+c-b)(b+c-a)s}{2(a+b-c)}}$....(9).

Similarly, $y=\sqrt{\dfrac{(a+b-c)(b+c-a)s}{2(a+c-b)}}$, $x=\sqrt{\dfrac{(a+b-c)(a+c-b)s}{2(b+c-a)}}$....(10, 11).

(9)+(10)+(11) gives $\sqrt{s}=\dfrac{2ab+2ac+2bc-a^2-b^2-c^2}{\sqrt{[2(a+b-c)(a+c-b)(b+c-a)]}}$....(12).

(12) in (9), (10), (11) gives

$x(b+c-a)=y(a+c-b)=z(a+b-z)=\frac{1}{2}(2ab+2ac+2bc-a^2-b^2-c^2)$.

Also solved by *J. M. BOORMAN*, *W. H. CARTER*, *C. C. CROSS*, *LESLIE L. LOCKE*, *COOPER D. SCHMITT*, *ELMER SCHUYLER*, *J. SCHEFFER*, *B. F. YANNEY*, *J. W. YOUNG*, and *M. A. GRUBER*.

GEOMETRY.

138. Proposed by JOHN M. HOWIE, Professor of Mathematics, The Nebraska State Normal, Peru, Neb.

K is the middle point of any chord AB of a given circle. CD and EF are any two chords passing through K. CF and ED intersect AB at M and N, respectively. Prove that KM equals KN.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.; P. S. BERG, B. Sc., Larimore, N. D.; and F. B. FILLMAN, Chester, Pa.

CASE I. N and M within the circle.
$\triangle DKN/\triangle CKM=DK.NK/CK.MK$....(1).
$\triangle EKN/\triangle FKM=EK.NK/FK.MK$....(2).
$\triangle CKM/\triangle EKN=CM.CK/EN.EK$....(3).
Multiplying (1) by (3),

$$\frac{\triangle DKN}{\triangle EKN}=\frac{DK.NK.CM}{MK.EN.EK}=\frac{DN}{EN}....(4).$$

Multiplying (2) by (3),

$$\frac{\triangle CKM}{\triangle FKM}=\frac{NK.CM.CK}{EN.FK.MK}=\frac{CM}{FM}....(5).$$